

Understanding the Gear Cube and how to create your own.

A work in progress...

Concepts:

Bevel Gears -

- Gears that rotate on non-parallel, yet intersecting axes

- Gear ratios

- Gear terminology including pitch and pitch circle

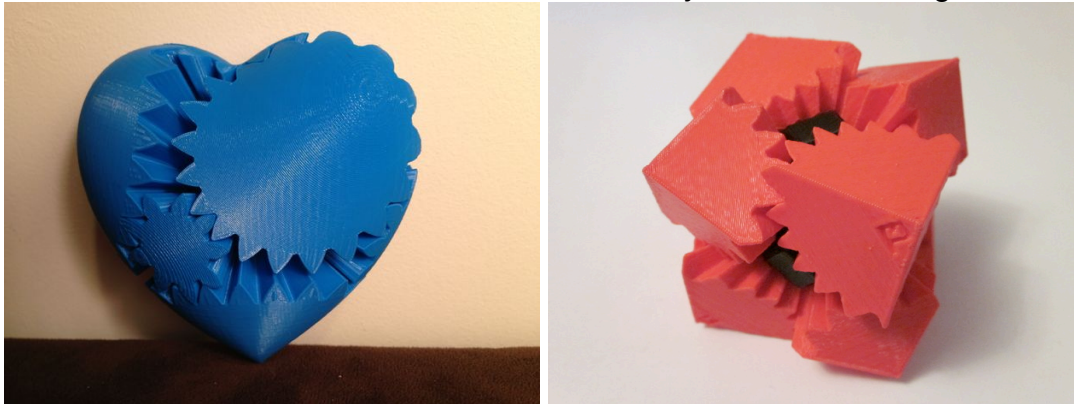
Trigonometry -

- finding the axes of rotation

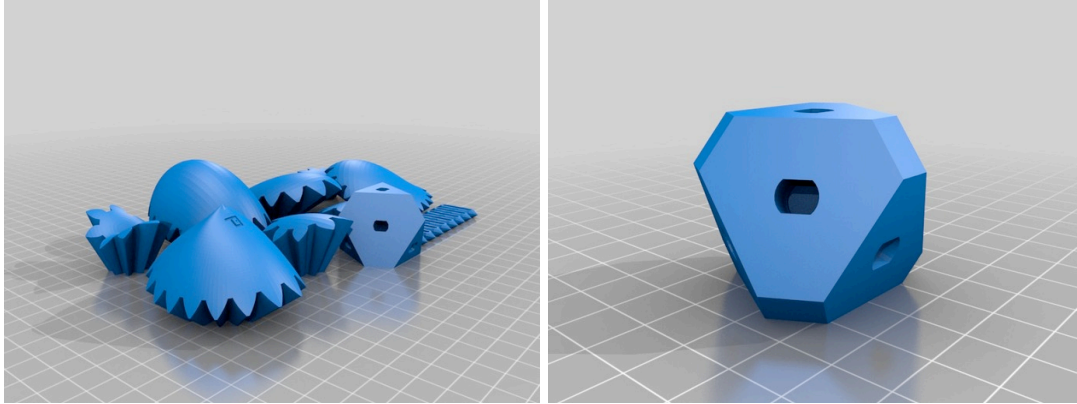
- calculating the pitch circle radius

I am not an engineer or a mathematician, but for years I have been fascinated with gears. As a result, I have written a number programs that produce different gear shapes including a modified involute gear, elliptical chainrings, logarithmic spirals and cardioids. Lately, I have been giving thought to how to make my own Gear Cube or Gear Heart or other solid shape.

Here are the Gear Heart and Gear Cube, both by Emmett on Thingiverse:

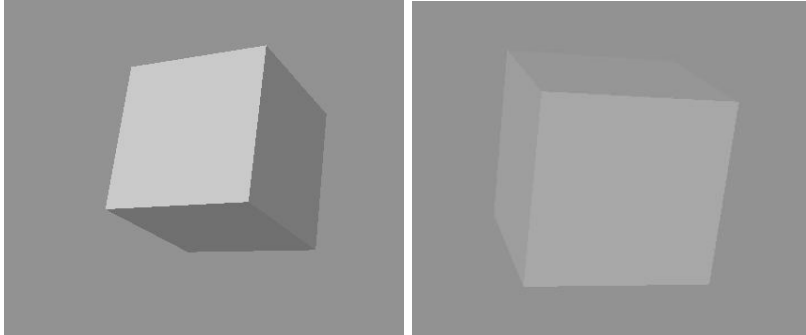


Both of the above assemblies, despite the different outward appearance of the heart and the cube, are actually functionally similar. In each case, the outer gears rotate around a core. Below are pictures that show the core. On the left is the core and gears for the Heart Gear and on the right is the core of the Gear Cube.



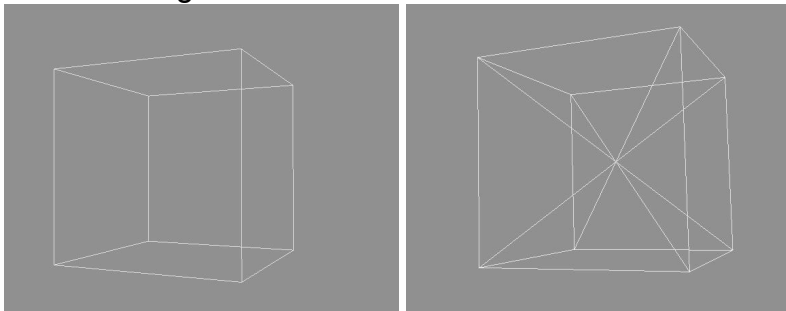
Getting started:

Let's look at a simple cube. What do we know about it? How many faces does it have? How many corners? How many edges?

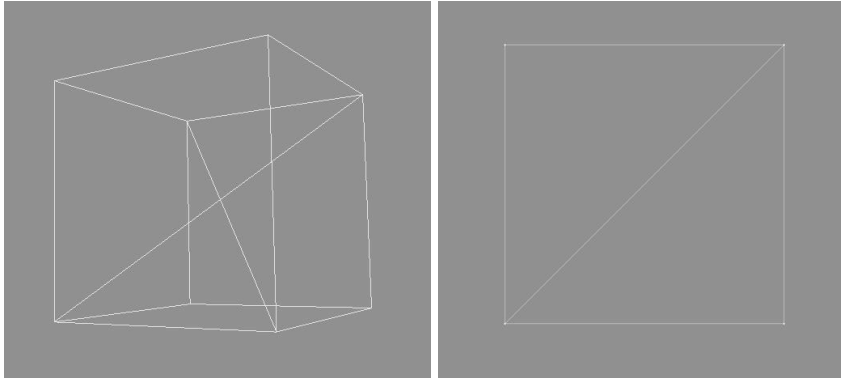


A cube is comprised of 6 square faces. It has 8 corners and 12 edges. And given that the cube is seemingly defined by 90 degree angles, it should be easy to figure out the axes of rotation and the conical angles of each gear, right? Not so fast...

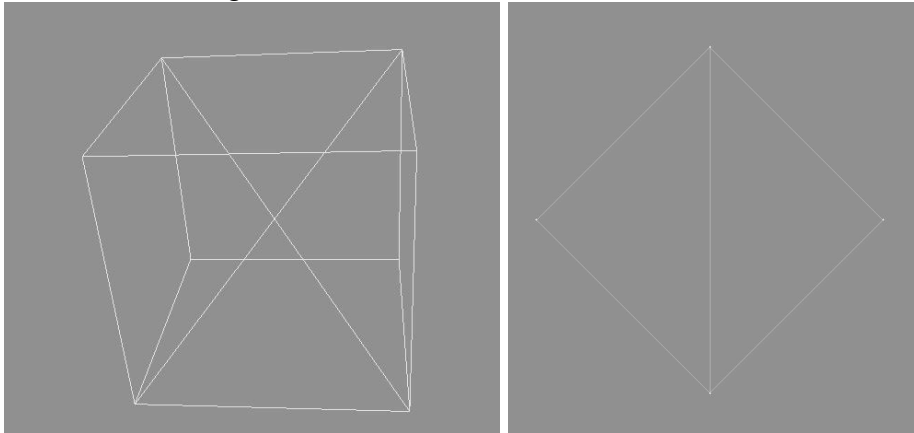
It might be easier if we look at a wireframe of a cube. Below left is a wireframe of a cube and below right is a wireframe of a cube that includes the 4 axes of rotation:



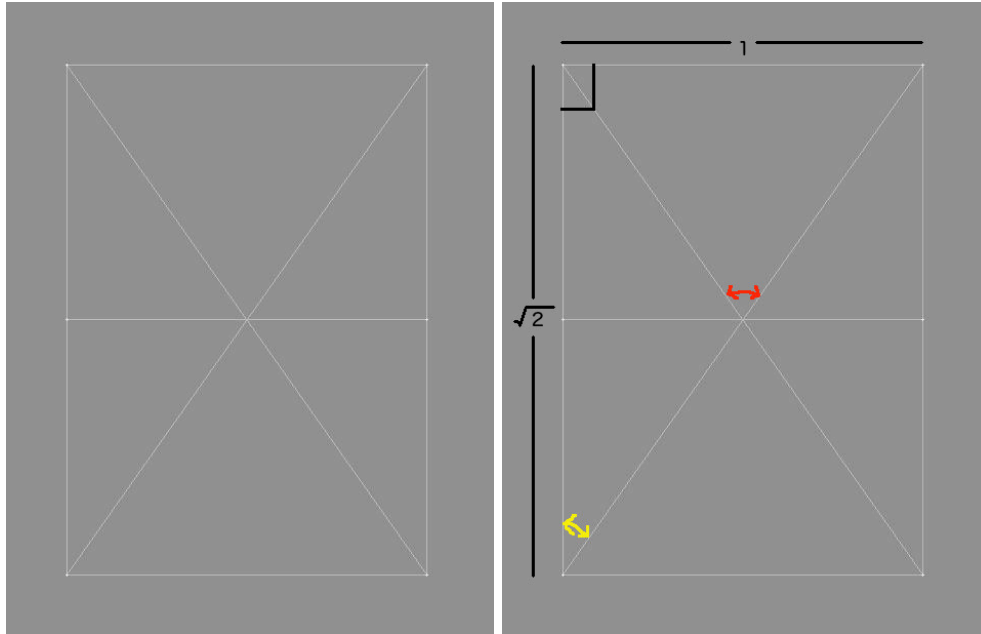
At first glance, it still may appear that the axes of rotation are perpendicular to each other. To get a better view, let's focus on two of the axes. Below is a picture of our cube with only 2 of the axes. On the left is a perspective view and on the right is a view from the right side:



From the side view, it becomes clear that the plane, upon which the axes of rotation reside, is at a 45 degree angle and it runs from the bottom front of the cube to the top back of the cube. To get a better view of the axes of rotation, let's rotate our cube 45 degrees so that the plane on which the axes of rotation resides is now vertical. Below on the left shows the rotated cube from perspective and on the right shows the rotated cube from the right:



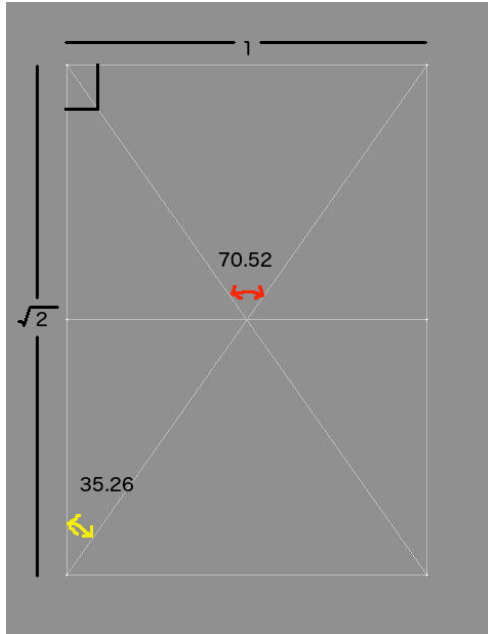
To get an even clearer view, let's look at it straight on from the front:



From the view above, it starts to become apparent that the angle of intersection (shown in red) between the axes of rotation is not perpendicular. So, how do we calculate the angle of intersection? First, we need to ascribe units to our cube. If we agree that our cube is one unit long by one unit wide by one unit tall, then we can begin to figure out the angle of intersection.

The first step in our calculations is to determine the height of our rotated cube. This is done by employing the Pythagorean Theorem ($A^2 + B^2 = C^2$). Given that our sides are a unit length of 1, then $1^2 + 1^2 = C^2$. $C^2 = 2$ and $C =$ the square root of 2.

Knowing the height of the rotated cube is the square root of 2, then we can calculate the angle of one of the axis of rotation. This is shown in yellow above. Remember that the tangent of an angle equals opposite over adjacent. And, if we know the length of the opposite and adjacent sides, we use ArcTangent to find the angle. In this case, we calculate the ArcTan of $1/\sqrt{2}$, which results in an angle of 35.2643 degrees. And since the crossing axis is also the same angle, we can simply double that angle to obtain an angle of intersection of 70.5287 degrees:

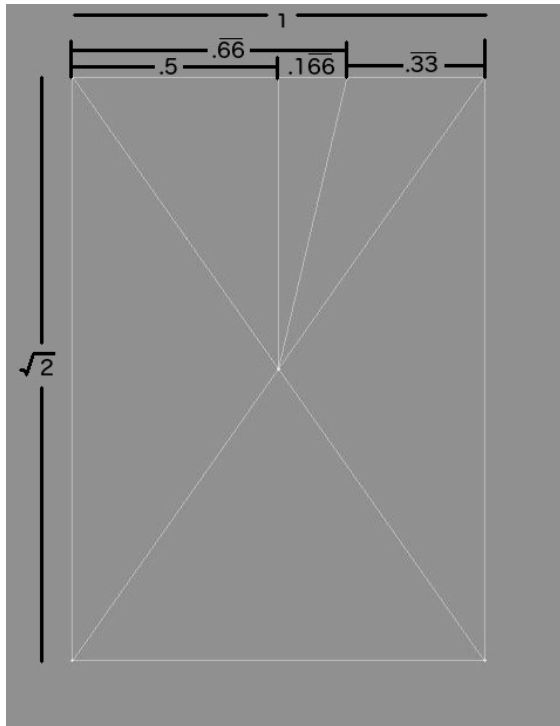


Now that we know the angle of intersection between the axes of rotation, we need to think about gear ratios for a moment. A gear ratio of 1:1 (pronounced “one-to-one”) would mean that two gears (one driving the other) are the same size and that turning one of the gears once would result in the other gear turning one time. A gear ratio of 2:1 (“two-to-one”) would mean that two gears are of different size (one exactly twice the size of the other) and that turning one of the gears once would result in the other gear turning two times.

For the purpose of this project, there are a number of possible gear ratios that could work. I am simply choosing a 2:1 ratio. Specifically, I will choose gears of 18 and 9 teeth.

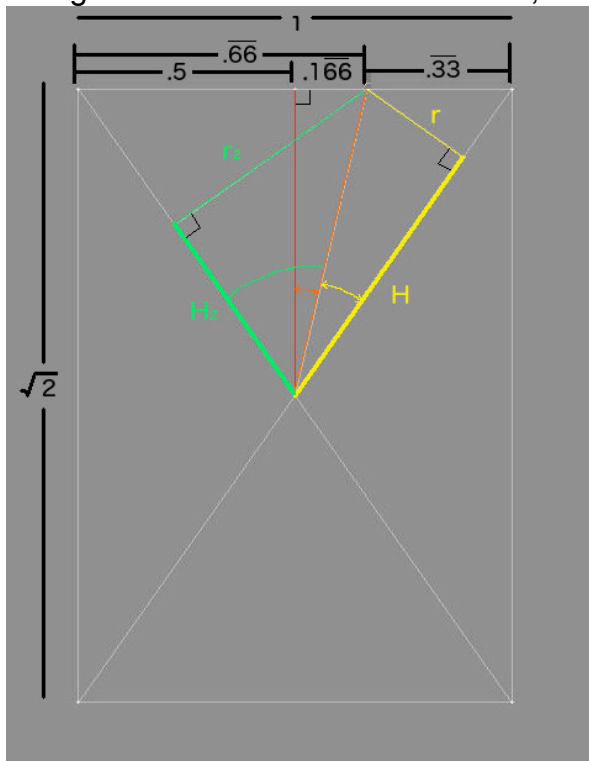
In order for our two gears to mesh together, they both need to be of the same pitch or teeth per inch. So, how are we going to calculate the ideal size of our gears?

We need to continue to divide the drawing of our cube. Thinking about a 2:1 ratio, let’s divide the upper line of our cube like this:



So, you may ask, why is there a mark at .66? Let's continue to think about a gear ratio of 2:1 for a moment. 66:33 is a two to one ratio. We simply need to divide the line into segments with two-thirds on one side and one-third on the other.

Using the measurements from above, we continue to mark up our drawing:



In the drawing above, the orange line segment represents the angle where the two gears are going to meet at their pitch circles. The yellow line (r) represents the radius of the pitch circle of the smaller gear. The green line (r2) represents the radius of the pitch circle of the larger gear. We want to solve for r and r2.

First, we are going to solve the orange line and the orange angle. Solving the orange again uses the Pythagorean Theorem. $(1/6)^2 + ((\sqrt{2}/2))^2 = \text{OrangeLine}^2$ or $\text{OrangeLine} = 0.72648$. To solve the OrangeAngle, we say $\text{OrangeAngle} = \text{ArcTangent}$ of opposite over adjacent or $\text{OrangeAngle} = \text{ArcTan}(1/6)/((\sqrt{2}/2))$ or 13.262676 degrees.

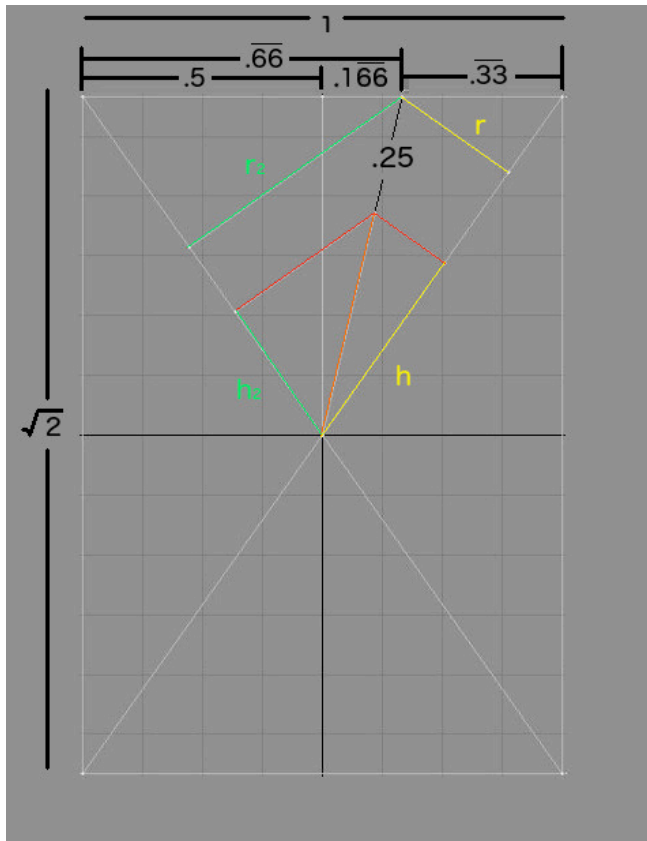
Next, we'll solve for the yellow angle. We already know that the OrangeAngle plus the YellowAngle = 35.2643 degrees. So, 35.2643 degrees minus OrangeAngle = the YellowAngle of 22.00162 degrees. We can now solve for the yellow line (r) by multiplying the hypotenuse (OrangeLine) by the sine of YellowAngle(22.001624 degrees) or $\text{YellowLine} = 0.72648 \times \sin(22.001624)$. This means that $r = 0.27216$

Switching to the green side, we know that GreenAngle = 35.2643 degrees plus 13.262676 degrees or GreenAngle = 48.526976. And, again, we use the sine of that angle multiplied by the hypotenuse to determine the GreenLine or $\text{GreenLine} = 0.72648 \times \sin(48.526976)$, which means that $r2 = 0.544327$.

The astute will notice that we didn't actually need to calculate r2. We simply did this to verify that r2 was, in fact, twice as big as r. This makes sense. With the 2:1 gearing that is used in this example, one gear will be twice as big as the other, thus the radius of the large gear will be exactly double the radius of the smaller gear.

Finally, we will solve for H and H2. To solve for H, we multiply the hypotenuse (OrangeLine) by the cosine of YellowAngle(22.001624 degrees) which looks like $H = 0.72648 \times \cos(22.001624)$ or $H = 0.673572$. Similarly, to solve for H2, we multiply the hypotenuse (OrangeLine) by the cosine of GreenAngle(48.526976 degrees) which looks like $H2 = 0.72648 \times \cos(48.526976)$ or $H2 = 0.481123$.

Next, we need to think of how thick we want our gears. Let's say we want our gears to be .25 of a unit thick. Observe the drawing below:

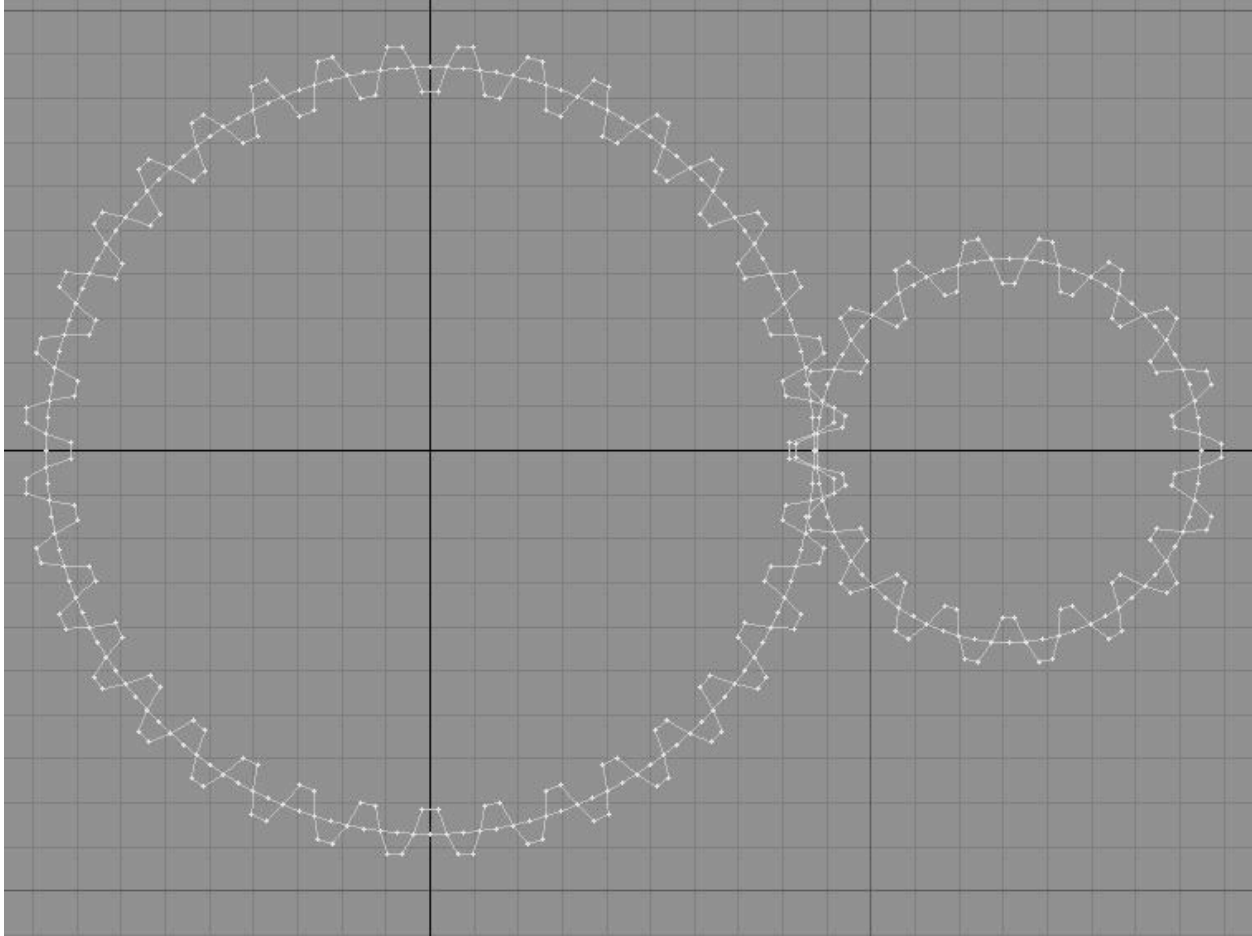


We have reduced the height of our original orange line by .25 units. And from this reduced height, we have drawn lines parallel to r and r_2 . Subtracting .25 from the original OrangeLine, we get ShortOrangeLine = 0.47648 units. And since the angles have remained unchanged, we can calculate h and h_2 .

To solve for h , we multiply the hypotenuse (NewOrangeLine) by the cosine of YellowAngle(22.001624 degrees) or $h = 0.47648 \times \cos(22.001624)$. The result is that h equals 0.441779. The same process is used to solve h_2 but using the green angle, meaning $h_2 = 0.47648 \times \cos(48.526976)$ or $h_2 = 0.315557$.

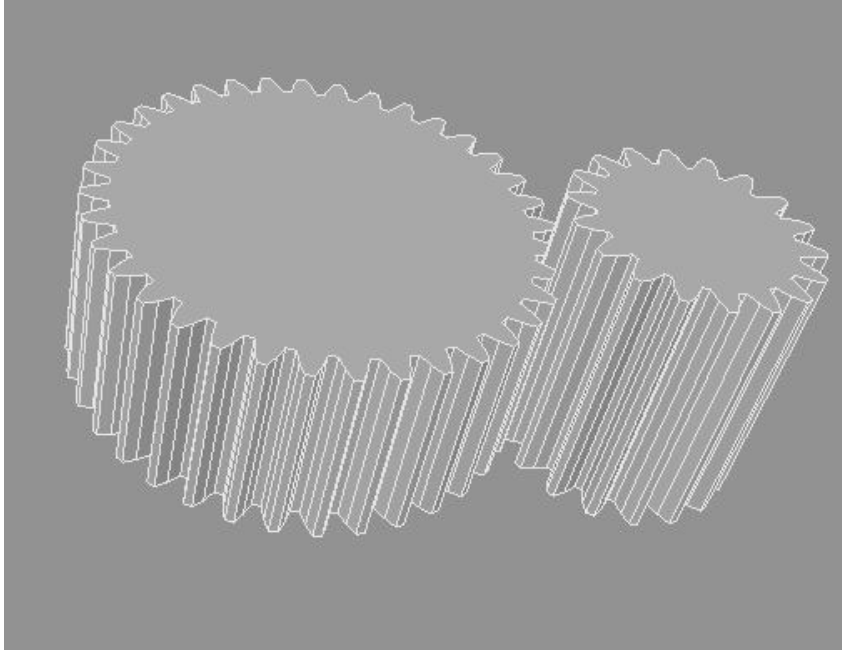
Believe it or not, we are almost done with our calculations! We just need to calculate our gear size. And since we know the radii of the gears, we can figure out how many teeth per unit we need so the gears will be the correct size. Starting with the smaller gear, we know that the radius is 0.27216. Using $2\pi r$, we arrive at a circumference of 1.71003. We also know that the smaller gear has 18 teeth, so we divide 1.71003 by 18 to derive a pitch of 0.0950. Some gear generating programs want to know how many teeth per unit, which is obtained by dividing 1 by the pitch, in this case $1/0.095$, which comes to 10.52612 teeth per unit. This will be the same pitch used by both gears.

Below is a picture of 2 gears, both of which are 10.52612 teeth per unit:

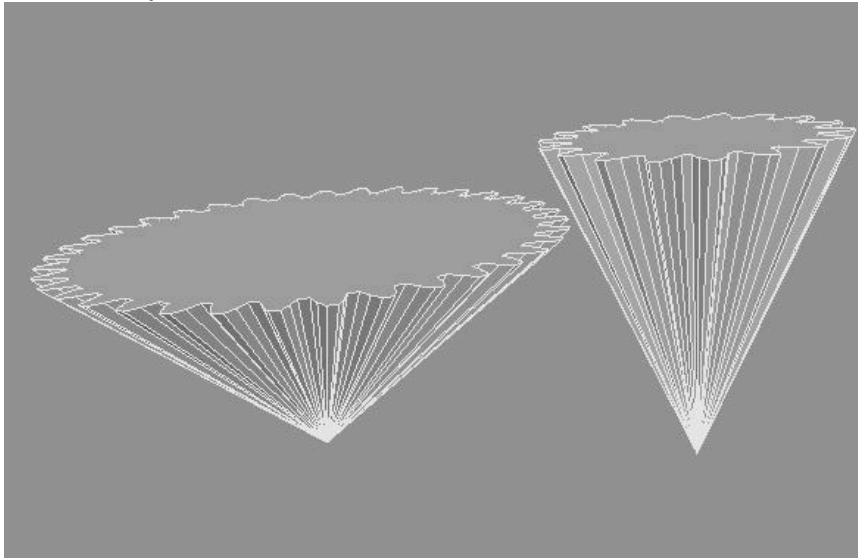


In the photo above, there is a gear of 36 teeth on the left and a gear of 18 teeth on the right. The gears are shown with concentric circles. The circle on the left has a radius of r_2 and the circle on the right has a radius of r . In other words, we were successful in exactly matching our gears to the radii we had calculated earlier. The gears were created in Lightwave 6.0 using a plugin I wrote to create gears. When we are finished, these gears will be available for download (include path to Thingiverse or Tinkercad)

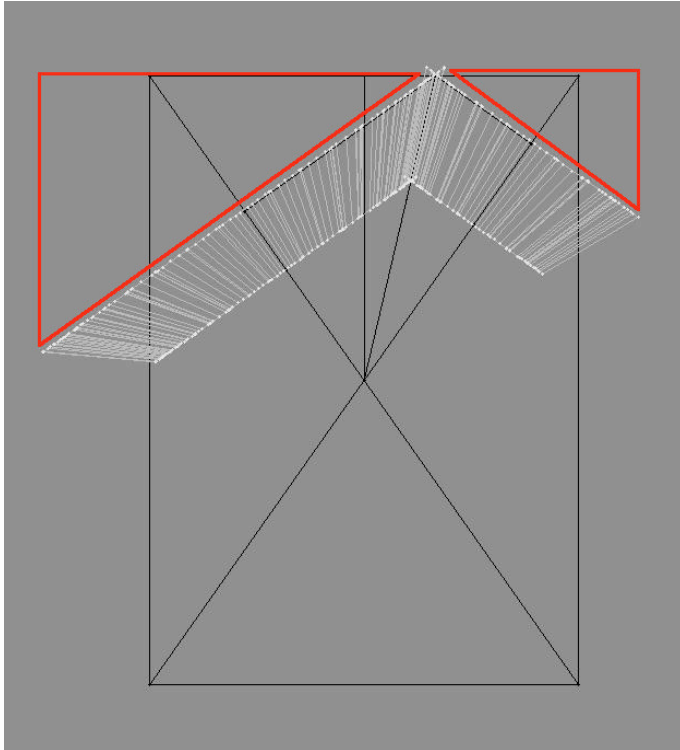
We are now going to set our gears to a height of zero and extrude to the correct height. The smaller gear needs to be extruded to a height of H and the larger gear need to be extruded to a height of H_2 . At this point, we will have 2 thick gears, but they aren't bevel gears yet. See below:



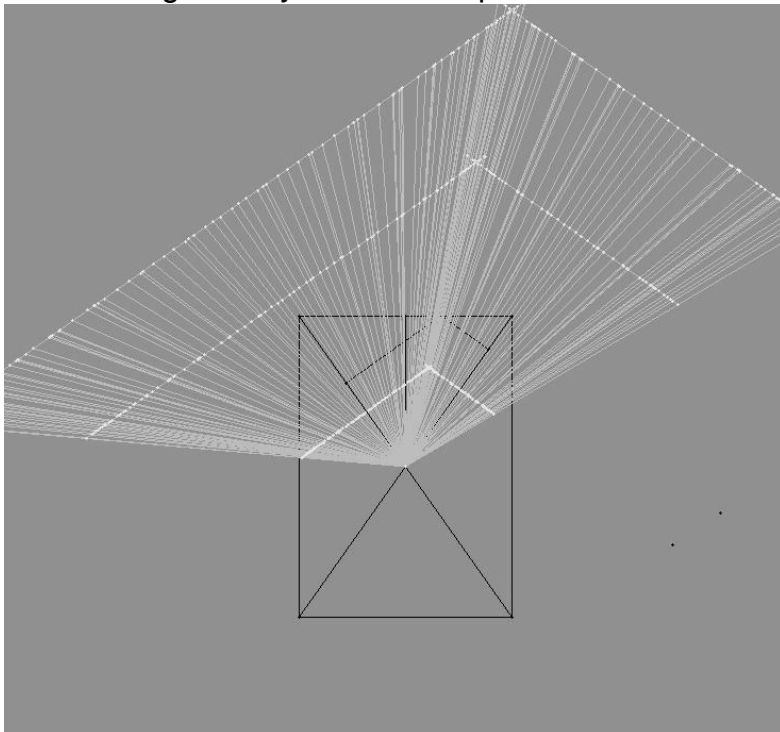
So, how do we turn these two gears into beveled gears? One easy way is to select all the points on the bottom of each gear and set them all to 0,0,0 which results in gears like in the photo below:



At this point, we could cut the bottom off and rotate them in position. And, the gears would be functional. However, we wouldn't have any material on the outside from which to make our cube or external solid. Look at the photo below:



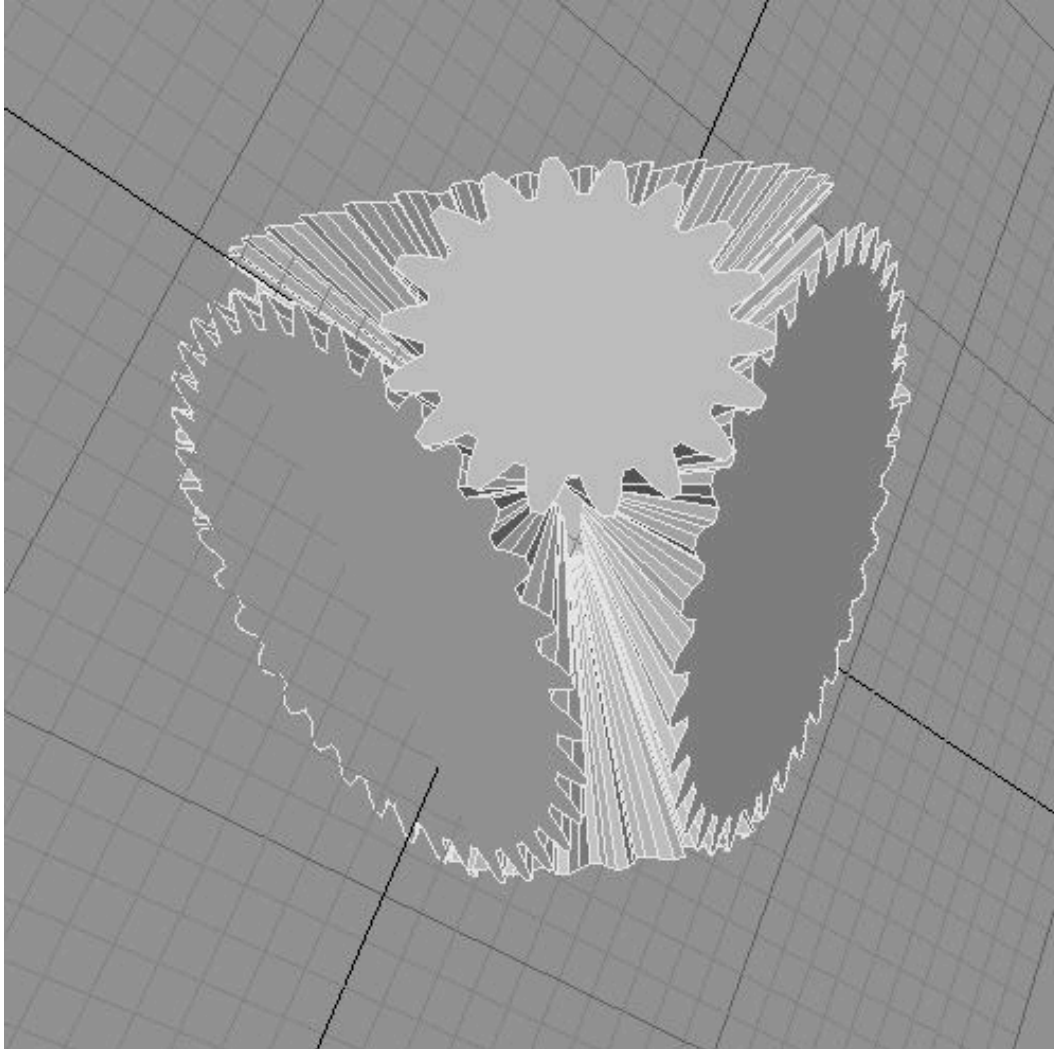
With the gears cut off and rotated in position, they will properly rotate around their axis of rotation and mesh with each other. Great! But, we still need more material out to where the red example lines are and beyond. Now, one cool thing about triangles and conical sections, is that when scaled up, a slice at a given elevation will retain the same scale. What? No matter how much scale up our gears, the slices that we need will keep their same geometry. Look at the picture below:



One can see that even increasing the scale of the bevel gear by 200 or 300%, all of the lines maintain their parallelism. One of my goals was to make the outer limits of this structure larger than needed, so that one could then sculpt it down to any desired shape.

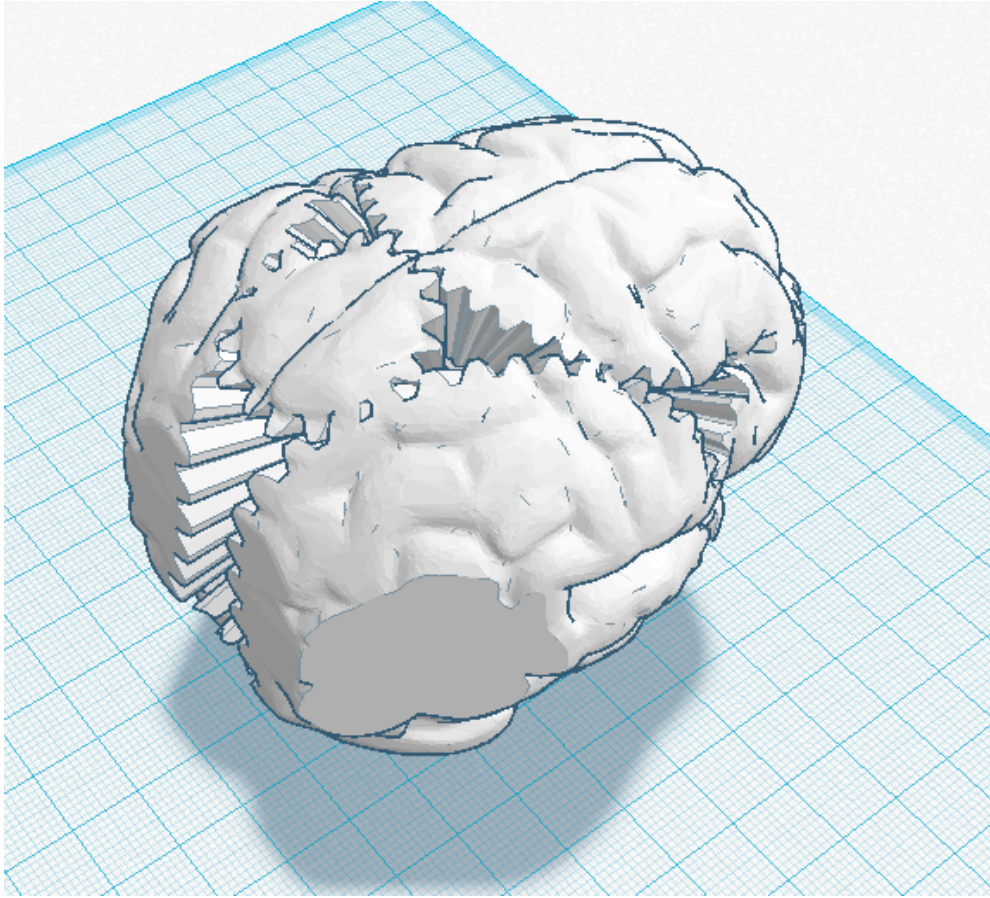
(Include a boolean step for creating the hole where the pins go)
(Include a step for creation of core)

By taking the now oversized gears and rotating them into the correct positions around the core, we get this:



The photo above shows an oversized set of meshing gears. Importing this into Tinkercad allows us to easily modify the shape of the gears.

The picture below is the first attempt to shape the gears into a brain shape:



Tinkercad is a little zealous about grouping things into solids.

